Dynamics II: rotation

L. Talley SIO 210 Fall, 2011

- DATES:
  - Oct. 24: second problem due
  - Oct. 24: short info about your project topic
  - Oct. 31: mid-term
  - Nov. 14: project due

- Rotation definitions
- Centrifugal force
- Coriolis force
- Inertial motion

- READING:
  - DPO: Chapter 7.2.3, 7.5.1
  - Stewart chapter 7.6, 9.1
  - Tomczak and Godfrey chapter 3, pages 29-35

Class project or paper

- Paper: choose 1 "classic" paper and one very recent paper (which does not have to be as comprehensive as a classic paper). How to choose? You might use the textbook to find either a modern or classic paper on a topic that is of interest to you. Email me with questions if you wish.

- JOA project: make sure you have a computer and operating system that allow you to install JOA before signing up for this.
- You will be the "test" team for these examples.
- (a) Write down impressions and advice for the manual as you install and learn to use JOA.
- (b) Choose a geographic region and work through data sets for that region: make a topographic map, plot profiles of properties vs. depth and vs. some other parameter, make a vertical section of a property versus depth and versus density if possible
- (c) write a short description of the section – what features do you observe and what are they, based at a minimum on the textbook

Rotating coordinates

- The Earth is rotating. We measure things relative to this "rotating reference frame".

- Quantity that tells how fast something is rotating:

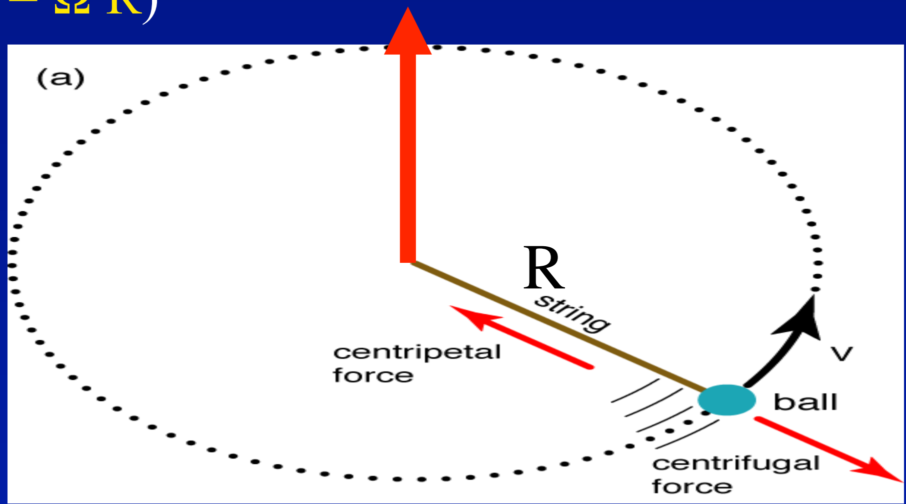
  Angular speed or angular velocity Ω = angle/second

  360° is the whole circle, but express angle in radians (2π radians = 360°)

  For Earth: 2π / 1 day = 2π / 86,400 sec = 0.707×10^{-4} /sec

  Also can show $\Omega = v/R$ where v is the measured velocity and R is the radius to the axis of rotation (therefore $v = \Omega R$)

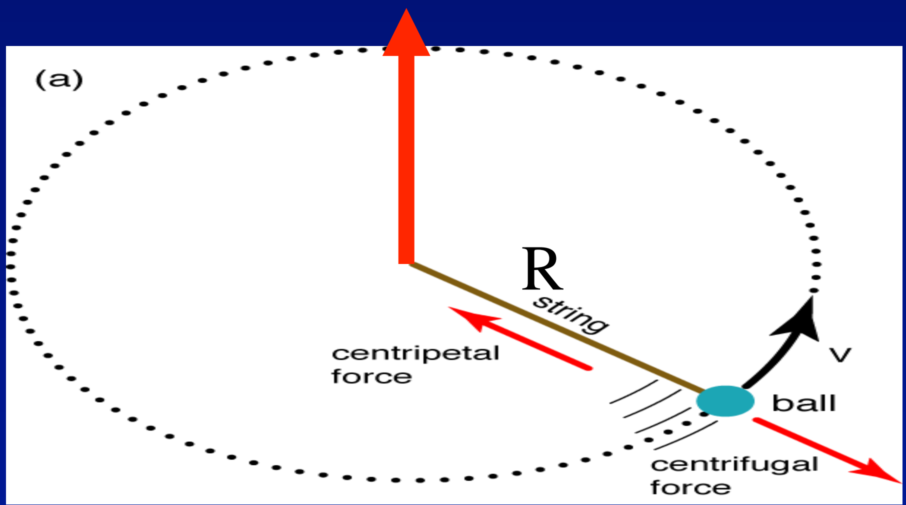
At home: calculate speed you are traveling through space if you are at the equator (radius of earth is about 6371 km), then calculate it at 30N as well. (Do some geometry to figure out the distance from the axis at 30N.)

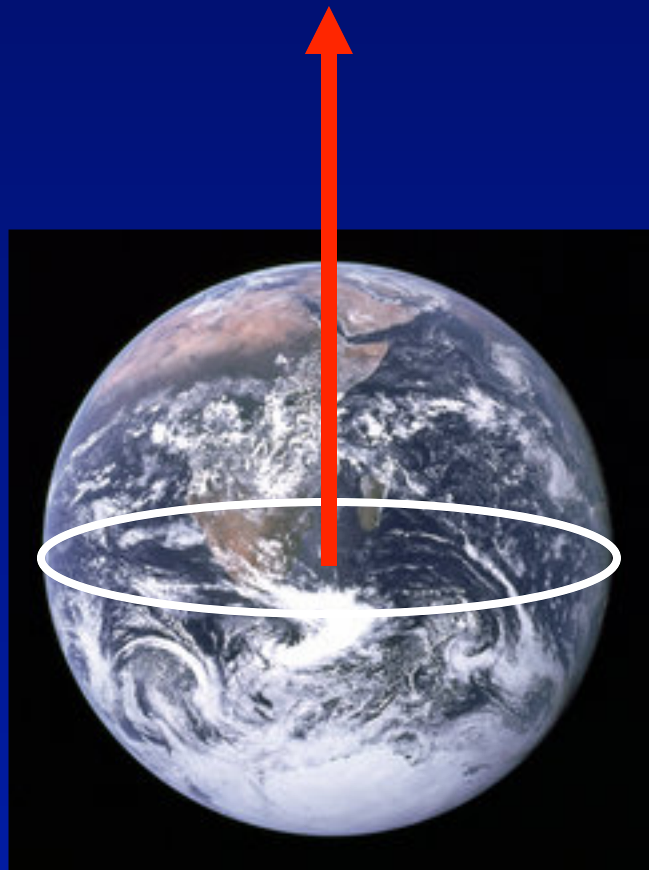


Rotating coordinates

- Vector that expresses direction of rotation and how fast it is rotating:

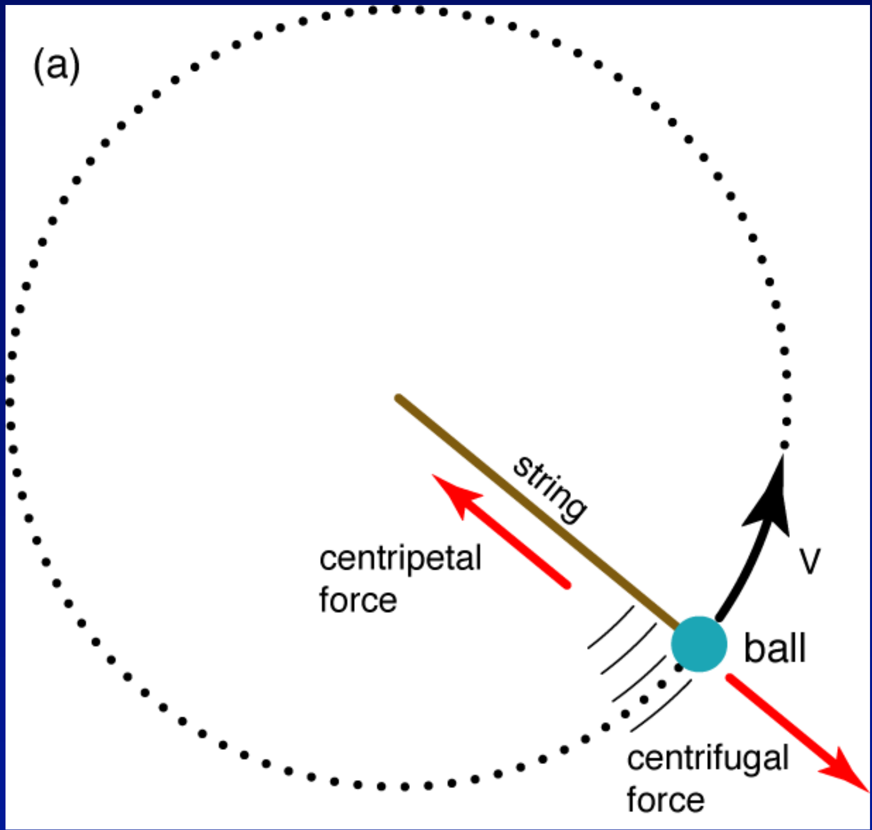
  vector pointing in direction of thumb using right-hand rule, curling fingers in direction of rotation





Centripetal and Centrifugal forces

(now looking straight down on the rotating plane)



Centripetal force is the actual force that keeps the ball "tethered" (here it is the string, but it can be gravitational force)

Centrifugal force is the pseudo-force (apparent force) that one feels due to lack of awareness that the coordinate system is rotating or curving

centrifugal acceleration = $\Omega^2 R$ (outward) (Units of m/sec^2)

Effect of centrifugal force on ocean and earth

Centrifugal force acts on the ocean and earth. It is pointed outward away from the rotation axis.

Therefore it is maximum at the equator (maximum radius from axis) and minimum at the poles (0 radius).

$\Omega = 0.707 \times 10^{-4}$ /sec

At the equator, R ~ 6380 km so $\Omega^2 R = .032\ m/s^2$

Compare with gravity = 9.8 m/s^2

Centrifugal force should cause the equator to be deflected (0.032/9.8) x 6380 km = 21 km outward compared with the poles. (i.e. about 0.3%)

Effect of centrifugal force on ocean and earth

Radius:

Equatorial 6,378.135 km

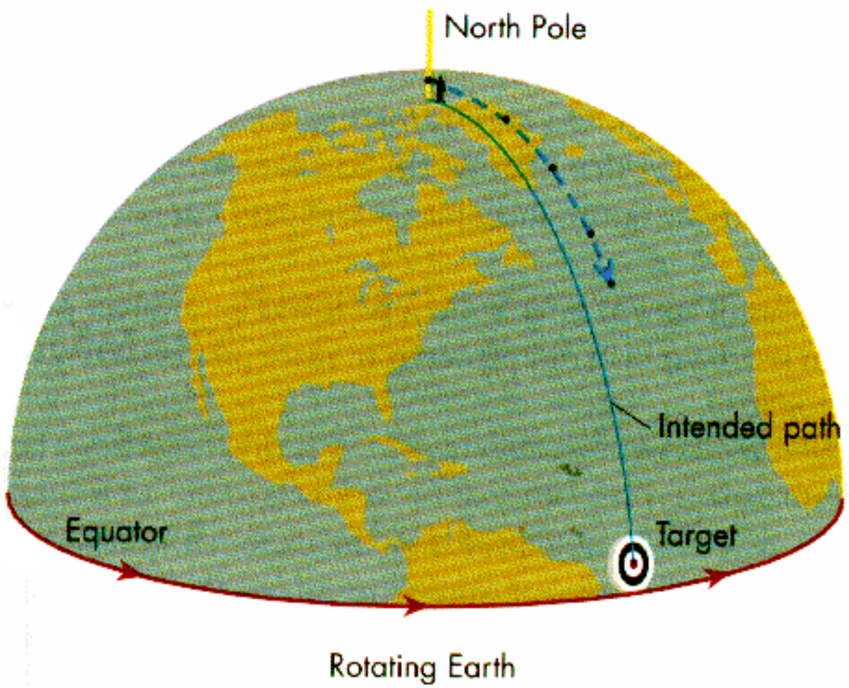
Polar 6,356.750 km

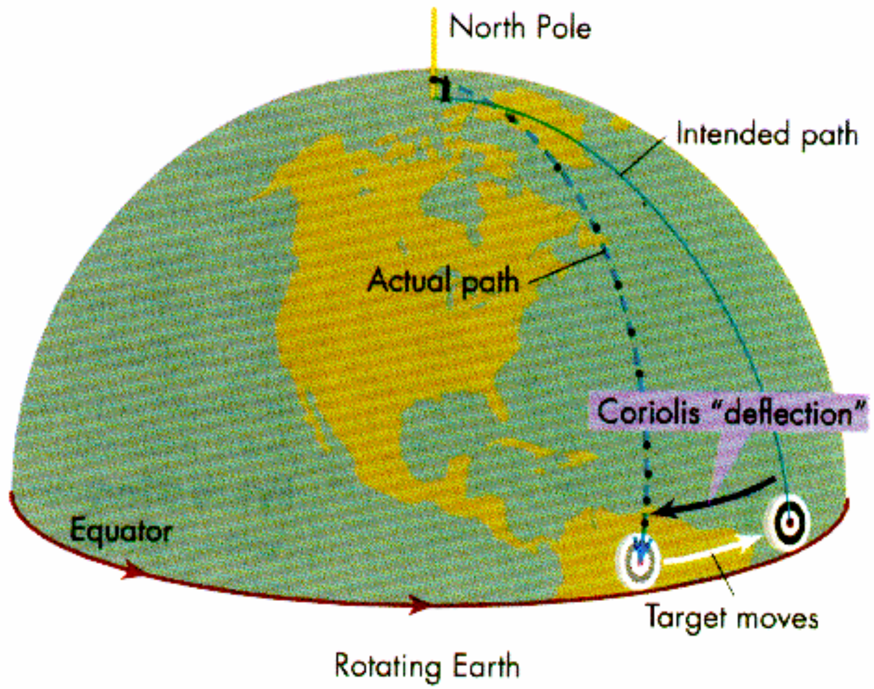
Mean 6,372.795 km

(From wikipedia entry on Earth)

The ocean is not 20 km deeper at the equator, rather the earth itself is deformed!

We bury the centrifugal force term in the gravity term (which we can call "effective gravity"), and ignore it henceforth. Calculations that require a precise gravity term should use subroutines that account for its latitudinal dependence.





Coriolis effect

Inertial motion:

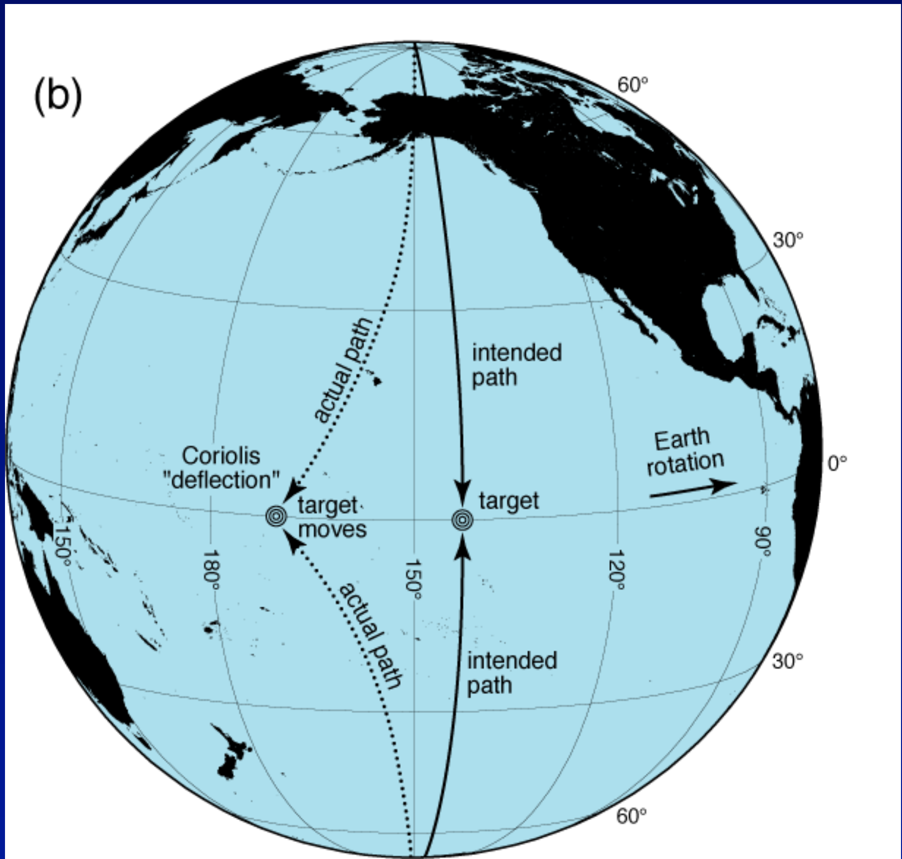
motion in a straight line relative to the fixed stars

Coriolis effect:

apparent deflection of that inertially moving body just due to the rotation of you, the observer.

Coriolis effect deflects bodies (water parcels, air parcels) to the right in the northern hemisphere and to the left in the southern hemisphere

Coriolis force



Additional terms in x, y momentum equations, at latitude φ
(horizontal motion is much greater than vertical)

x-momentum equation:
$-2\ \Omega \sin\varphi\ v \quad = \ -f\ v$

y-momentum equation:
$2\ \Omega \sin\varphi\ u \quad = \ f\ u$

$f = 2\ \Omega \sin\varphi$ is the "Coriolis parameter". It depends on latitude (projection of total Earth rotation on local vertical)

Complete force (momentum) balance with rotation

Three equations:

Horizontal (x) (west-east)

acceleration + advection + Coriolis =

pressure gradient force + viscous term

Horizontal (y) (south-north)

acceleration + advection + Coriolis =

pressure gradient force + viscous term

Vertical (z) (down-up)

acceleration +advection (+ neglected very small Coriolis) =

pressure gradient force + effective gravity

(including centrifugal force) + viscous term

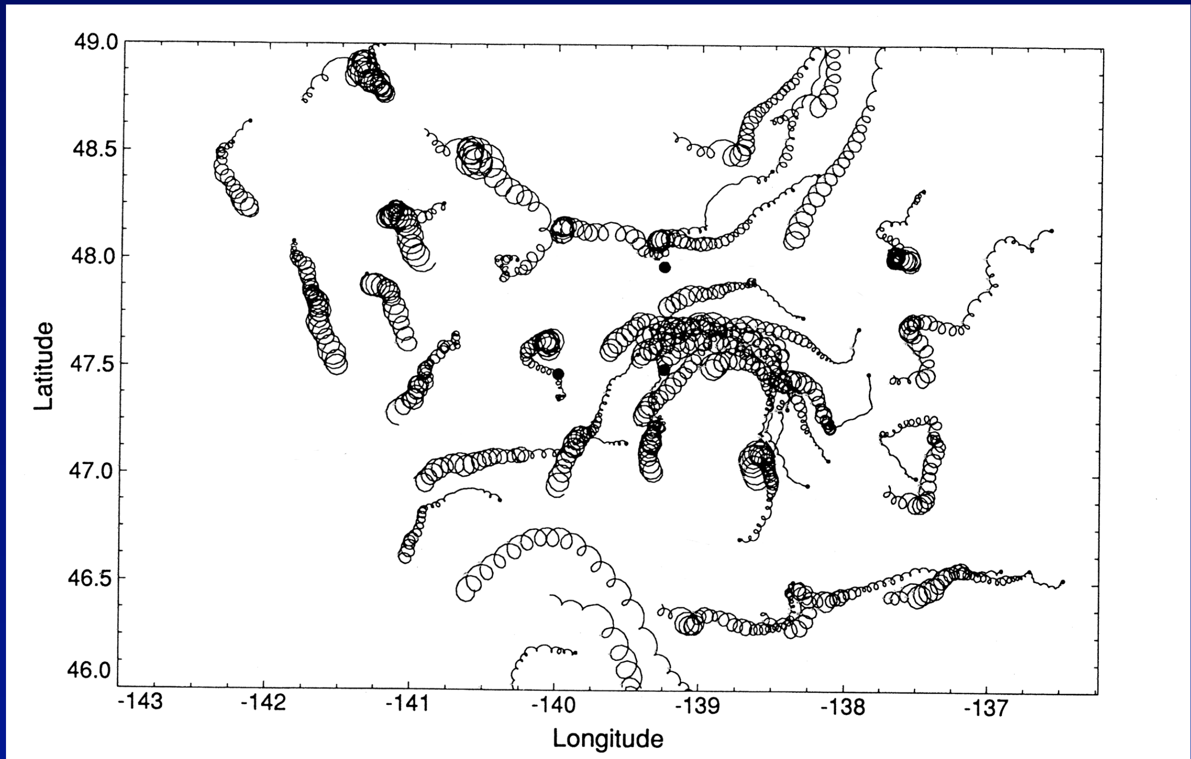
Final equations of motion (momentum equations in Cartesian coordinates)

x: $\partial u/\partial t + u\, \partial u/\partial x + v\, \partial u/\partial y + w\, \partial u/\partial z - fv =$
$- (1/\rho)\partial p/\partial x + \partial/\partial x(A_H \partial u/\partial x) +$
$\partial/\partial y(A_H \partial u/\partial y) + \partial/\partial z(A_V \partial u/\partial z)$ (7.11a)

y: $\partial v/\partial t + u\, \partial v/\partial x + v\, \partial v/\partial y + w\, \partial v/\partial z + fu =$
$- (1/\rho)\partial p/\partial y + \partial/\partial x(A_H \partial v/\partial x) +$
$\partial/\partial y(A_H \partial v/\partial y) + \partial/\partial z(A_V \partial v/\partial z)$ (7.11b)

z: $\partial w/\partial t + u\, \partial w/\partial x + v\, \partial w/\partial y + w\, \partial w/\partial z$ (+ neglected small Coriolis) =
$- (1/\rho)\partial p/\partial z - g + \partial/\partial x(A_H \partial w/\partial x) +$
$\partial/\partial y(A_H \partial w/\partial y) + \partial/\partial z(A_V \partial w/\partial z)$ (7.11c)

Coriolis in action in the ocean: Observations of Inertial Currents



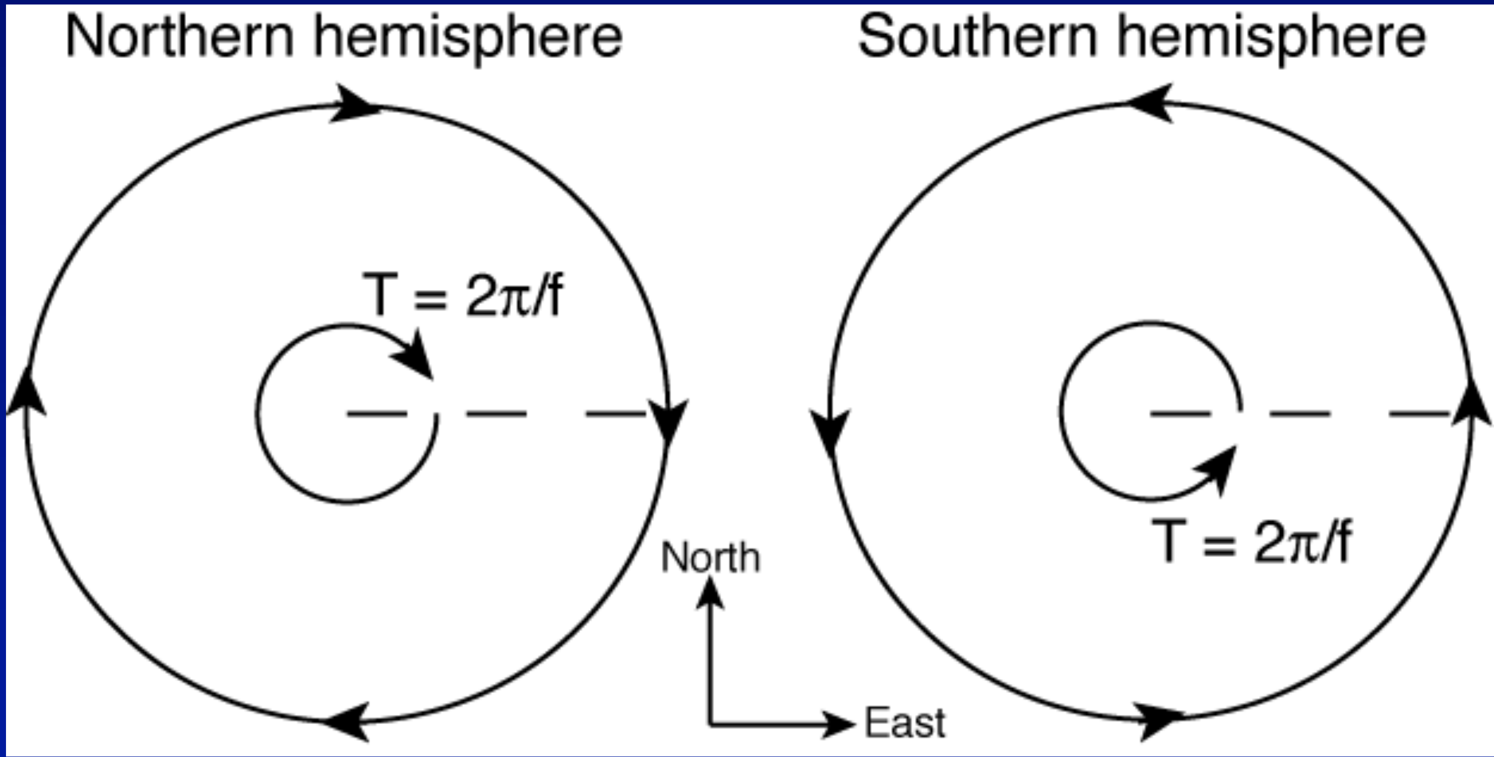
D'Asaro et al. (1995)

DPO Fig. 7.4

- Surface drifters in the Gulf of Alaska during and after a storm.
- Note the corkscrews - drifters start off with clockwise motion, which gets weaker as the motion is damped (friction)

Inertial currents

Balance of Coriolis and acceleration terms: push the water and it turns to the right (NH), in circles



DPO Fig. S7.8a

Inertial currents: force balance

Three APPROXIMATE equations:

Horizontal (x) (west-east)

acceleration + Coriolis = 0

Horizontal (y) (south-north)

acceleration + Coriolis = 0

Vertical (z) (down-up) (NOT IMPORTANT) (hydrostatic)

0 = pressure gradient force + effective gravity

That is:

x: $\partial u/\partial t - fv = 0$

y: $\partial v/\partial t + fu = 0$