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FORTRAN PROGRAMS FOR THE EQUATION OF STATE OF MOIST AIR

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CONTENTS

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ABSTRACT. A review of the physical variables and laws concerning the equation of state of moist air is presented. Related Fortran programs are given.

1. The gas laws.

In the following, reference is made to an ideal gas, according to the classical model by Bernoulli (1738). Lists of symbols, constants and formulas are supplied in the Appendix. SI units (IAPSO, 1979) are adopted.

A gas is described by a finite number of properties, such as pressure (p), temperature (T) and volume (V), called variables of state. Gay-Lussac's law, or Charles' law (1781) states that the gas volume, at constant pressure, linearly increases with temperature; if θ is the Celsius temperature:

$$V(\theta) \ = \ V(0)\,(1 + \mathrm{a}\,\theta) \ = \ V(0)\,\mathrm{a}\,T \ ,$$

where T_o=1/a defines the difference between the thermodinamic and the Celsius temperature scales. Boyle's law (1660) states that, at constant temperature,

$$p\,V \ = \ \text{constant} \ .$$

The amount of substance of a system having as many elementary entities as the number N_A (Avogadro's number) of atoms present in 12 kg of carbon-12 is a basic SI unit called kilomole (symbol: kmol); the elementary entities (atoms, molecules, ions, ...) must be specified. A kilomole of gas at standard temperature and pressure (0 °C, 1013.25 hPa) has the standard molar volume $V_M(0)$. The molar mass of a gas is defined by the ratio $M=m/n$ between its mass and the number n of moles.

In general, the variables of state are related by the equation of state:

$$p \ = \ \rho\,R\,T \ ,$$

where the constant of the gas $R=R^*/M$ is a function of the universal gas constant R^* and of the molar mass.

If the system involved is a mixture of ideal gases, the equation of state holds for each of them separately (Dalton's law). In this case, at constant temperature T, each gas occupies the total volume V and contributes to the total pressure p with its partial pressure p_i. Otherwise, each gas can be compressed to the pressure p in a partial volume V_i mantaining the same temperature; in this case, according to Boyle's law $p_i V = p V_i$ or, in other words, the fractional volumes F_i, the fractional pressures and the fractional numbers of moles are the same:

$$F_i \ = \ V_i / V \ = \ p_i / p \ = \ n_i / n \ .$$

The density of the mixture is the sum of the densities of each component, given by the equation of state:

$$\rho_i \ = \ (R^* T)^{-1}\, M_i\, p_i \ .$$

Introducing the fractional volumes, it is $p_i = pF_i$ and

$$\rho = (R^* T)^{-1} \underline{M} p \ ,$$

where the average molar mass of the mixture

$$\underline{M} = \Sigma F_i M_i$$

has been defined. A mixture of gases has therefore the same equation of state of a single ideal gas of average molar mass $\underline{M}$.

2. The equation of state of moist air.

The earth's atmosphere is a mixture of gases which can be described with a high accuracy by the laws of thermodynamics for ideal gases. Water vapour is usually considered apart from dry air, since water can change state at environmental temperatures with relevant energy transfers.

Dry air is well mixed in the troposphere; the composition of the U.S. Standard Atmosphere (NOAA, 1976), considered constant up to a height of 87 km, is given in the following table. The average molar mass is M_a=28.9644 kg/kmol.

Gas species	Molar mass M_i /(kg/kmol)	Fractional volume F_i
N_2	28.0134	0.78084
O_2	31.9988	0.209476
Ar	39.948	0.00934
CO_2	44.00995	0.000314
Ne	20.183	0.00001818
He	4.0026	0.00000524
Kr	83.80	0.00000114
Xe	131.30	0.000000087
CH_4	16.04303	0.000002
H_2	2.01594	0.0000005

The moist air is a mixture of dry air, average molar mass M_a, and water vapour, molar mass M_v=18.0153 kg/kmol. If p is the moist air (atmospheric) pressure and e the partial pressure of the water vapour, the average molar mass of (moist) air, weighting with the fractional pressures, is

$$M = M_a \,(p- e)/p + M_v \, e/p = M_a \,(1 - (1 - \varepsilon \,)\, e/p) \ ,$$

where the molar ratio $\varepsilon = M_v / M_a$= 0.62198 has been defined. The equation of state for the moist air mixture gives therefore the atmospheric density

$$\rho = (R^* T\,)^{-1} M_a \, p \,(1 - (1 - \varepsilon\,)\, e/p) \ .$$

A "virtual temperature"

$$T_v = T /(1 - (1 - \varepsilon\,)\, e/p) \ ,$$

representing the temperature of a dry atmosphere with the same density

$$\rho = (R^* T_v)^{-1} M_a \, p$$

of the moist one at T, is usually introduced. T_v is slightly greater than T to compensate for the density loss produced by the sostitution of dry air molecules with lighter water vapour molecules. In practice, T_v is replaced by the so called "adjusted virtual temperature" $T_v' = CT_v$, where the compressibility factor of moist air C, slightly smaller than one, accounts for the fact that a higher density is requested to a real gas in order to have the ideal pressure. A constant value $C = 0.9995$ at 10 °C, 1100 hPa, 50 % relative humidity is commonly assumed. Using the adjusted virtual temperature (the temperature of a real moist atmosphere behaving like an ideal dry air) and the dry air constant $R=R^*/M_a= 287.053$ J.kg^{-1}.K^{-1}, the equation of state for the earth's atmosphere is:

$$\rho = (R\, T_v')^{-1}\, p \ .$$

3. The atmospheric humidity.

The water vapour amount in the atmosphere is described by a set of variables:

(i) the water vapour density $d_v= m_v/V$ or "absolute humidity";
(ii) the mixing ratio $r = m_v/m_a$ between the water vapour and dry air masses;
(iii) the specific water vapour mass $q = m_v/(m_a+m_v)$ or "specific humidity";
(iv) the partial pressure e, e_w indicating the equilibrium or saturation pressure over liquid water;
(v) the ratio between the water vapour mass and the corresponding mass at saturation $u = m_v/m_w$ or "relative humidity";
(vi) the percentual relative humidity $U = 100\, u$.

Relations between these variables are given in the Appendix.

The saturation water vapor pressure $e_w(T)$ over liquid water at different air temperatures is given by the Goff-Gratch (1946) formula (13), reported also in the Smithsonian Meteorological Tables. This formula can be used for temperatures as low as -60 °C, with an error smaller than 3% (Detwiler, 1983).

Direct measurements of water vapour in the atmosphere are not performed in practical meteorology. The conventional method is to measure the air (or "dry bulb") Celsius temperature (θ) and the "wet bulb" temperature (θ_w) by means of the aspirating psychrometer. At the equilibrium, the temperature lapse ($\theta - \theta_w$), produced by the heat transfer from the wet bulb to the evaporating water (psychrometric effect), is a function of air dryness and of the air velocity around the bulb. The water vapor pressure deficit from saturation is related to the psichrometric temperature lapse by Sprung's formula (19); the Ferrel coefficient A (18) is used for ventilations in the range 4-10 m/s. Relative humidity is then computed from e, e_w by means of (15). The approximate formula $u = e/e_w$ is sometime used.

The rates of change of the air density with respect to p, θ and U are easily computed from the equation of state. Typical values are reported in the following table.

	0 °C	20 °C	
$\Delta\rho/\Delta p = 1/(RT)$	+1.3	+1.2	g.m^{-3}/hPa
$\Delta\rho/\Delta\theta = -p/(RT^2)$	- 4.7	- 4.1	g.m^{-3}/°C (at 1013.25 hPa)
$\Delta\rho/\Delta U = -3.8 \times 10^{-3}\, e_w\, \Delta\rho/\Delta p$	- 0.03	- 0.11	g.m^{-3}/% (at 1013.25 hPa)

Table 1 gives a set of air density values at standard atmospheric pressure, for temperatures between -10 and 40 °C and relative humidity from 0 to 100 %. Only decimal digits are listed, so that, for example, density at 20 °C and 50 % is 1.1994 kg/m^3. **Tables 2, 3** refer to a 60 % relative humidity and to atmospheric pressures from 950 to 1050 hPa.

The water vapour pressure and the corresponding absolute humidity at the saturation, for temperatures between -29 and 50 °C, are listed in **Table 4**.

4. The Fortran programs.

The Fortran subroutine AIRES(p,θ,θ_w,U,K,e,e_w,r,q,d_v,ρ), (AIR Equation of State, pages 11-12) computes the air density, the water vapour pressures, the mixing ratio, the specific and the absolute humidity at given atmospheric pressure, temperature and (i) wet bulb temperature or (ii) relative humidity. Input/output parameters are controlled by the argument K as follows:

K=1 input : p, θ, θ_w ;
output : U, e, e_w, r, q, d_v, ρ .

K=2 input : p, θ, U ;
output : e, e_w, r, q, d_v, ρ .

The function WSP(T), page 12, is used to compute $e_w(\theta)$ by means of the Goff-Gratch formula (13); the water pressure e is computed according to (18,19) if K=1, according to (17) if K=2; the mixing ratios r and r_w (at saturation) are given by (12), T_v, T_v' by (22,23), ρ by (24), q and d_v by (8,9). If K=1, U is computed according to (14,16). Notice that AIRES yields r and q multiplied by 10^3.

Program AIRCOMP (page 13) asks the input and computes the output arguments above by calling AIRES and displays the results.

REFERENCES

DETWILER A. (1983): "Extrapolation of the Goff-Gratch formula for vapor pressure of liquid water at temperatures below 0 °C", J. Climate Appl. Meteorol., 22, 503-504.

IAPSO (1979): "SUN report on the use in Physical sciences of the ocean of the Systeme International d'Unites (SI) and related standards for symbols and terminology", IUGG Publ. Off., France, 56 pp.

NOAA (1976): "U.S. Standard Atmosphere, 1976", NOAA-S/T76-1562.

5. APPENDIX

(A) LIST OF SYMBOLS

Symbol	units	variable
V	m^3	volume
F		fractional volume
n	mol	number of moles
n_a	mol	number of moles of dry air
n_v	mol	number of moles of water vapour
m_a	kg	mass of dry air
m_v	kg	mass of water vapour
m_w	kg	mass of water vapour at saturation
M	kg/kmol	molar mass
p	Pa	(total) atmospheric pressure
e	Pa	water vapour pressure
e_w	Pa	water vapour pressure at saturation
ρ	kg/m^3	(moist) air density
d_v	kg/m^3	water vapour density (absolute humidity)
r		mixing ratio
r_w		mixing ratio at saturation
q		specific humidity
u		relative humidity
U	%	percentual relative humidity
T	K	air thermodynamic temperature
T_v	K	virtual temperature
T_v'	K	adjusted virtual temperature
θ	°C	air (dry bulb) Celsius temperature
θ_w	°C	wet bulb Celsius temperature
A	$°C^{-1}$	Ferrel coefficient

(B) CONSTANTS

$V_M(0)$	$= 22.4136\ m^3/kmol$	standard molar volume
N_A	$= 6.022169 \times 10^{26}\ kmol^{-1}$	Avogadro's number
R^*	$= 8.31432 \times 10^3\ J.kmol^{-1}.K^{-1}$	universal gas constant
M_a	$= 28.9644\ kg/kmol$	molar mass of dry air
M_v	$= 18.0153\ kg/kmol$	molar mass of water vapour
ε	$= M_v / M_a = 0.62198$	molar ratio
R	$= R^*/M_a = 287.053\ J.kg^{-1}.K^{-1}$	constant of dry air
T_o	$= 273.15\ K$	origin of Celsius temperature
T_1	$= 273.16\ K$	triple point of water
C	$= 0.9995$	compressibility factor of moist air.

(C) FORMULAS

(1)	m_a	$= n_a M_a$
(2)	m_v	$= n_v M_v$
(3)	ρ	$= (m_a + m_v)/V$
(4)	d_v	$= m_v/V$
(5)	r	$= m_v/m_a$
(6)	q	$= m_v/(m_a + m_v)$
(7)	r	$= q/(1 - q)$
(8)	q	$= r/(1 + r)$
(9)	d_v	$= \rho q$
(10)	e/p	$= n_v/(n_a + n_v) = r/(r + \varepsilon)$
(11)	e	$= p\, r/(r + \varepsilon)$
(12)	r	$= \varepsilon\, e/(p - e)$
(13)	$\mathrm{Log}_{10}\, e_w(T)$	$= 10.79574(1 - T_1/T) - 5.02800\, \mathrm{Log}_{10}(T/T_1) +$ $+ 1.50475 \times 10^{-4} (1 - 10^{-8.2969 (T/T_1 - 1)})$ $+ 0.42873 \times 10^{-3} (10^{4.76955 (1 - T_1/T)} - 1) + 0.78614$
(14)	u	$= m_v/m_w = r/r_w$
(15)	u	$= (e/e_w)(p - e_w)/(p - e)$
(16)	U	$= 100\, u$
(17)	e	$= e_w\, u/(1 - (1 - u)\, e_w/p)$
(18)	A	$= 0.000660\, (1 + 0.00115\, \theta_w/°\mathrm{C})\quad °\mathrm{C}^{-1}$
(19)	e	$= e_w(T_w) - \mathrm{A}\, p\, (\theta - \theta_w)$
(20)	T_v	$= T/(1 - (1 - \varepsilon)\, e/p)$
(21)	T_v	$= T(1 - p/e_w - u)/(1 - p/e_w - \varepsilon u)$
(22)	T_v	$= T(1 + r/\varepsilon)/(1 + r)$
(23)	T_v'	$= C\, T_v$
(24)	ρ	$= p/(R\, T_v')$

°C	0 %	10 %	20 %	30 %	40 %	50 %	60 %	70 %	80 %	90 %	100 %
-10	3420	3419	3418	3416	3415	3413	3412	3410	3409	3408	3406
-9	3370	3368	3367	3365	3363	3362	3360	3359	3357	3356	3354
-8	3319	3318	3316	3314	3313	3311	3309	3308	3306	3304	3303
-7	3269	3267	3266	3264	3262	3260	3258	3257	3255	3253	3251
-6	3220	3218	3216	3214	3212	3210	3208	3206	3204	3202	3200
-5	3170	3168	3166	3164	3162	3160	3158	3156	3154	3152	3150
-4	3121	3119	3117	3115	3112	3110	3108	3106	3104	3101	3099
-3	3073	3070	3068	3066	3063	3061	3058	3056	3054	3051	3049
-2	3025	3022	3019	3017	3014	3012	3009	3007	3004	3001	2999
-1	2977	2974	2971	2968	2966	2963	2960	2957	2955	2952	2949
0	2929	2926	2923	2920	2917	2914	2911	2909	2906	2903	2900
1	2882	2879	2876	2872	2869	2866	2863	2860	2857	2854	2850
2	2835	2832	2828	2825	2822	2818	2815	2811	2808	2805	2801
3	2789	2785	2781	2778	2774	2771	2767	2763	2760	2756	2753
4	2743	2739	2735	2731	2727	2723	2719	2715	2712	2708	2704
5	2697	2693	2688	2684	2680	2676	2672	2668	2664	2660	2655
6	2651	2647	2642	2638	2634	2629	2625	2620	2616	2612	2607
7	2606	2601	2597	2592	2587	2582	2578	2573	2568	2564	2559
8	2561	2556	2551	2546	2541	2536	2531	2526	2521	2516	2511
9	2517	2511	2506	2501	2495	2490	2484	2479	2474	2468	2463
10	2473	2467	2461	2455	2450	2444	2438	2432	2427	2421	2415
11	2429	2422	2416	2410	2404	2398	2392	2386	2380	2374	2368
12	2385	2379	2372	2365	2359	2352	2346	2340	2333	2327	2320
13	2342	2335	2328	2321	2314	2307	2300	2293	2286	2280	2273
14	2299	2291	2284	2277	2269	2262	2255	2247	2240	2233	2226
15	2256	2248	2240	2232	2225	2217	2209	2201	2194	2186	2178
16	2214	2205	2197	2189	2180	2172	2164	2155	2147	2139	2131
17	2172	2163	2154	2145	2136	2127	2118	2110	2101	2092	2084
18	2130	2120	2111	2101	2092	2083	2073	2064	2055	2046	2036
19	2088	2078	2068	2058	2048	2038	2028	2019	2009	1999	1989
20	2047	2036	2026	2015	2004	1994	1983	1973	1963	1952	1942
21	2006	1995	1983	1972	1961	1950	1939	1928	1917	1906	1895
22	1965	1953	1941	1929	1918	1906	1894	1882	1871	1859	1847
23	1925	1912	1899	1887	1874	1862	1849	1837	1825	1812	1800
24	1885	1871	1858	1844	1831	1818	1805	1792	1778	1766	1753
25	1845	1831	1816	1802	1788	1774	1760	1746	1732	1719	1705
26	1805	1790	1775	1760	1745	1730	1715	1701	1686	1672	1657
27	1766	1750	1734	1718	1702	1686	1671	1655	1640	1625	1610
28	1727	1710	1693	1676	1659	1643	1626	1610	1594	1578	1562
29	1688	1670	1652	1634	1617	1599	1582	1565	1547	1530	1514
30	1650	1631	1612	1593	1574	1556	1537	1519	1501	1483	1465
31	1611	1591	1571	1551	1531	1512	1493	1473	1454	1435	1417
32	1573	1552	1531	1510	1489	1468	1448	1428	1408	1388	1368
33	1536	1513	1490	1468	1446	1425	1403	1382	1361	1340	1319
34	1498	1474	1450	1427	1404	1381	1358	1336	1313	1292	1270
35	1461	1435	1410	1386	1361	1337	1313	1290	1266	1243	1220
36	1424	1397	1370	1344	1319	1293	1268	1243	1219	1194	1170
37	1387	1359	1331	1303	1276	1249	1223	1197	1171	1145	1120
38	1350	1320	1291	1262	1233	1205	1177	1150	1123	1096	1070
39	1314	1282	1251	1221	1191	1161	1132	1103	1074	1046	1019
40	1278	1244	1212	1180	1148	1117	1086	1056	1026	996	967

TABLE 1. Air density $(\rho/(\text{kg/m}^3)-1) \times 10^4$ at 1013.25 hPa as a function of temperature and relative humidity.

°C	950	955	960	965	970	975	980	985	990	995	1000
-10	2574	2640	2707	2773	2839	2905	2971	3038	3104	3170	3236
-9	2526	2592	2658	2724	2790	2856	2922	2988	3054	3120	3186
-8	2478	2544	2609	2675	2741	2806	2872	2938	3004	3069	3135
-7	2430	2496	2561	2627	2692	2758	2823	2889	2954	3019	3085
-6	2383	2448	2513	2578	2644	2709	2774	2839	2905	2970	3035
-5	2336	2401	2466	2531	2596	2661	2726	2791	2856	2921	2986
-4	2289	2354	2418	2483	2548	2613	2677	2742	2807	2872	2936
-3	2242	2307	2371	2436	2500	2565	2629	2694	2758	2823	2887
-2	2196	2260	2325	2389	2453	2517	2582	2646	2710	2775	2839
-1	2150	2214	2278	2342	2406	2470	2534	2598	2662	2726	2790
0	2104	2168	2232	2296	2360	2423	2487	2551	2615	2679	2742
1	2059	2122	2186	2250	2313	2377	2440	2504	2567	2631	2695
2	2014	2077	2140	2204	2267	2330	2394	2457	2520	2584	2647
3	1969	2032	2095	2158	2221	2284	2347	2410	2474	2537	2600
4	1924	1987	2050	2113	2175	2238	2301	2364	2427	2490	2553
5	1879	1942	2005	2067	2130	2193	2255	2318	2381	2443	2506
6	1835	1897	1960	2022	2085	2147	2210	2272	2334	2397	2459
7	1791	1853	1915	1977	2040	2102	2164	2226	2288	2351	2413
8	1747	1809	1871	1933	1995	2057	2119	2181	2243	2305	2367
9	1703	1765	1827	1888	1950	2012	2074	2135	2197	2259	2321
10	1660	1721	1783	1844	1906	1967	2029	2090	2152	2213	2275
11	1616	1677	1739	1800	1861	1923	1984	2045	2107	2168	2229
12	1573	1634	1695	1756	1817	1878	1940	2001	2062	2123	2184
13	1530	1591	1652	1712	1773	1834	1895	1956	2017	2078	2139
14	1487	1547	1608	1669	1730	1790	1851	1912	1972	2033	2094
15	1444	1504	1565	1625	1686	1746	1807	1867	1928	1988	2049
16	1401	1462	1522	1582	1642	1703	1763	1823	1883	1944	2004
17	1359	1419	1479	1539	1599	1659	1719	1779	1839	1899	1959
18	1316	1376	1436	1496	1556	1615	1675	1735	1795	1855	1915
19	1274	1333	1393	1453	1512	1572	1632	1691	1751	1811	1870
20	1231	1291	1350	1410	1469	1529	1588	1648	1707	1766	1826
21	1189	1248	1308	1367	1426	1485	1545	1604	1663	1722	1782
22	1147	1206	1265	1324	1383	1442	1501	1560	1619	1678	1737
23	1105	1164	1222	1281	1340	1399	1458	1517	1576	1634	1693
24	1063	1121	1180	1239	1297	1356	1415	1473	1532	1591	1649
25	1021	1079	1137	1196	1254	1313	1371	1430	1488	1547	1605
26	978	1037	1095	1153	1211	1270	1328	1386	1445	1503	1561
27	936	994	1052	1111	1169	1227	1285	1343	1401	1459	1517
28	894	952	1010	1068	1126	1184	1241	1299	1357	1415	1473
29	852	910	967	1025	1083	1140	1198	1256	1314	1371	1429
30	810	867	925	982	1040	1097	1155	1212	1270	1327	1385
31	768	825	882	939	997	1054	1111	1169	1226	1283	1341
32	725	782	839	897	954	1011	1068	1125	1182	1239	1296
33	683	740	797	854	910	967	1024	1081	1138	1195	1252
34	640	697	754	810	867	924	981	1037	1094	1151	1208
35	597	654	711	767	824	880	937	993	1050	1107	1163
36	555	611	667	724	780	837	893	949	1006	1062	1119
37	512	568	624	680	737	793	849	905	961	1018	1074
38	468	525	581	637	693	749	805	861	917	973	1029
39	425	481	537	593	649	704	760	816	872	928	984
40	381	437	493	549	604	660	716	771	827	883	938

TABLE 2. Air density $(\rho/(\mathrm{kg/m^3})-1) \times 10^4$ at 60% relative humidity as a function of air temperature and pressure (950-1000 hPa).

°C	1000	1005	1010	1015	1020	1025	1030	1035	1040	1045	1050
-10	3236	3303	3369	3435	3501	3568	3634	3700	3766	3832	3899
-9	3186	3252	3318	3383	3449	3515	3581	3647	3713	3779	3845
-8	3135	3201	3267	3332	3398	3464	3529	3595	3661	3727	3792
-7	3085	3150	3216	3281	3347	3412	3478	3543	3609	3674	3740
-6	3035	3100	3166	3231	3296	3361	3427	3492	3557	3622	3687
-5	2986	3051	3116	3181	3246	3311	3376	3441	3506	3570	3635
-4	2936	3001	3066	3131	3195	3260	3325	3390	3454	3519	3584
-3	2887	2952	3016	3081	3145	3210	3274	3339	3404	3468	3533
-2	2839	2903	2967	3032	3096	3160	3224	3289	3353	3417	3482
-1	2790	2854	2919	2983	3047	3111	3175	3239	3303	3367	3431
0	2742	2806	2870	2934	2998	3061	3125	3189	3253	3317	3380
1	2695	2758	2822	2885	2949	3012	3076	3140	3203	3267	3330
2	2647	2710	2774	2837	2900	2964	3027	3090	3154	3217	3280
3	2600	2663	2726	2789	2852	2915	2978	3041	3105	3168	3231
4	2553	2616	2678	2741	2804	2867	2930	2993	3056	3119	3181
5	2506	2569	2631	2694	2756	2819	2882	2944	3007	3070	3132
6	2459	2522	2584	2647	2709	2771	2834	2896	2959	3021	3084
7	2413	2475	2537	2600	2662	2724	2786	2848	2911	2973	3035
8	2367	2429	2491	2553	2615	2677	2739	2801	2863	2925	2987
9	2321	2383	2444	2506	2568	2630	2691	2753	2815	2877	2938
10	2275	2337	2398	2460	2521	2583	2644	2706	2767	2829	2890
11	2229	2291	2352	2413	2475	2536	2597	2659	2720	2781	2843
12	2184	2245	2306	2367	2429	2490	2551	2612	2673	2734	2795
13	2139	2200	2261	2322	2382	2443	2504	2565	2626	2687	2748
14	2094	2154	2215	2276	2336	2397	2458	2519	2579	2640	2701
15	2049	2109	2170	2230	2291	2351	2412	2472	2533	2593	2654
16	2004	2064	2125	2185	2245	2305	2366	2426	2486	2546	2607
17	1959	2019	2079	2139	2200	2260	2320	2380	2440	2500	2560
18	1915	1975	2034	2094	2154	2214	2274	2334	2394	2453	2513
19	1870	1930	1990	2049	2109	2169	2228	2288	2348	2407	2467
20	1826	1885	1945	2004	2064	2123	2183	2242	2302	2361	2420
21	1782	1841	1900	1959	2019	2078	2137	2196	2256	2315	2374
22	1737	1796	1856	1915	1974	2033	2092	2151	2210	2269	2328
23	1693	1752	1811	1870	1929	1988	2046	2105	2164	2223	2282
24	1649	1708	1766	1825	1884	1942	2001	2060	2118	2177	2236
25	1605	1664	1722	1780	1839	1897	1956	2014	2073	2131	2190
26	1561	1619	1678	1736	1794	1852	1911	1969	2027	2085	2144
27	1517	1575	1633	1691	1749	1807	1865	1923	1982	2040	2098
28	1473	1531	1589	1647	1704	1762	1820	1878	1936	1994	2052
29	1429	1487	1544	1602	1660	1717	1775	1833	1890	1948	2006
30	1385	1442	1500	1557	1615	1672	1730	1787	1845	1902	1960
31	1341	1398	1455	1513	1570	1627	1684	1742	1799	1856	1914
32	1296	1354	1411	1468	1525	1582	1639	1696	1753	1811	1868
33	1252	1309	1366	1423	1480	1537	1594	1651	1708	1765	1821
34	1208	1264	1321	1378	1435	1491	1548	1605	1662	1718	1775
35	1163	1220	1276	1333	1389	1446	1503	1559	1616	1672	1729
36	1119	1175	1231	1288	1344	1401	1457	1513	1570	1626	1682
37	1074	1130	1186	1242	1299	1355	1411	1467	1523	1580	1636
38	1029	1085	1141	1197	1253	1309	1365	1421	1477	1533	1589
39	984	1040	1095	1151	1207	1263	1319	1375	1430	1486	1542
40	938	994	1050	1105	1161	1217	1272	1328	1384	1439	1495

TABLE 3. Air density $(\rho/(\mathrm{kg/m^3})-1) \times 10^4$ at 60% relative humidity as a function of air temperature and pressure (1000-1050 hPa).

θ/°C	e_w/hPa	d_v/(g/m^3)	θ/°C	e_w/hPa	d_v/(g/m^3)
-29	0.56	0.50	**11**	13.12	10.01
-28	0.61	0.54	**12**	14.02	10.66
-27	0.67	0.59	**13**	14.97	11.34
-26	0.74	0.65	**14**	15.98	12.06
-25	0.81	0.70	**15**	17.04	12.82
-24	0.88	0.77	**16**	18.17	13.62
-23	0.96	0.84	**17**	19.37	14.47
-22	1.05	0.91	**18**	20.63	15.36
-21	1.15	0.99	**19**	21.96	16.30
-20	1.25	1.07	**20**	23.37	17.28
-19	1.37	1.17	**21**	24.86	18.32
-18	1.49	1.26	**22**	26.43	19.41
-17	1.62	1.37	**23**	28.08	20.56
-16	1.76	1.48	**24**	29.83	21.76
-15	1.91	1.61	**25**	31.67	23.03
-14	2.08	1.74	**26**	33.61	24.35
-13	2.25	1.88	**27**	35.65	25.75
-12	2.44	2.03	**28**	37.79	27.21
-11	2.64	2.19	**29**	40.05	28.74
-10	2.86	2.36	**30**	42.43	30.34
-9	3.10	2.54	**31**	44.92	32.02
-8	3.35	2.74	**32**	47.55	33.78
-7	3.62	2.95	**33**	50.30	35.62
-6	3.91	3.17	**34**	53.20	37.55
-5	4.21	3.41	**35**	56.23	39.56
-4	4.54	3.66	**36**	59.42	41.67
-3	4.90	3.93	**37**	62.76	43.87
-2	5.27	4.22	**38**	66.26	46.17
-1	5.68	4.52	**39**	69.93	48.57
0	6.11	4.85	**40**	73.77	51.07
1	6.57	5.19	**41**	77.80	53.69
2	7.05	5.56	**42**	82.01	56.41
3	7.57	5.95	**43**	86.42	59.26
4	8.13	6.36	**44**	91.03	62.22
5	8.72	6.79	**45**	95.85	65.31
6	9.35	7.26	**46**	100.89	68.53
7	10.01	7.75	**47**	106.15	71.88
8	10.72	8.27	**48**	111.65	75.37
9	11.47	8.81	**49**	117.40	79.00
10	12.27	9.39	**50**	123.39	82.78

TABLE 4. Water vapour pressure and absolute humidity at saturation versus air temperature.

```
      SUBROUTINE AIRES(P,T,TW,U,K,E,EW,R,Q,D,RO)
C--------------------------------------- Franco Stravisi 1988
C  INPUT PARAMETERS:
C  P    atmospheric pressure /hPa ,
C  T    air temperature /°C .
C  INPUT/OUTPUT PARAMETERS controlled by  K=1/2 :
C  TW   wet bulb temperature /°C ,
C  U    percentual relative humidity /% ;
C  if  K = 1   TW is INPUT  and  U is OUTPUT ,
C  if  K = 2   U  is INPUT  and TW is neglected .
C  OUTPUT PARAMETERS:
C  E    water vapor pressure /hPa ,
C  EW   saturation water vapor pressure /hPa ,
C  R    1000 x mixing ratio ,
C  Q    1000 x specific humidity ,
C  D    absolute humidity /(g.m-3) ,
C  RO   moist air density /(kg.m-3) .
C
C  OUTPUT in COMMON /VIRTEM/:
C  TV   virtual temperature /°C ,
C  TVP  adjusted virtual temperature /°C .
C
C-------CONSTANTS:
C  EPS = 0.62198  molar ratio ,
C  RA  = 287.053  (J/kg)/K   dry air constant ,
C  C   = 0.9995   compressibility factor of moist air .
C------------------------------------------------------------
      COMMON/VIRTEM/TV,TVP
C
      EPS=0.62198
      RA=287.053
      C=0.9995
      TK=T+273.15
C---- Pressure /Pa
      PA=P*100.
C----
      EW=WSP(T)
      IF(K.EQ.2) GOTO 100
C---- K=1: computation of E (Sprung psycrometric formula)
C----      A is the Ferrel coefficient
      A= 6.60E-4*(1.+0.00115*TW)
      E=WSP(TW)-A*P*(T-TW)
      GOTO 200
  100 CONTINUE
C---- K=2: computation of E as a function of U
      UR=U/100
      E=UR*EW/(1.-(1.-UR)*EW/P)
C----
  200 CONTINUE
      R=EPS*E/(P-E)
      R1=R+1.
      RW=EPS*EW/(P-EW)
```

```
      IF(K.EQ.1) U=100*R/RW
      TV=TK*(1.+R/EPS)/R1
      TVP=C*TV
      RO=PA/(RA*TVP)
C---- R,Q,D times 1000:
      R=R*1000.
      Q=R/R1
      D=Q*RO
      RETURN
      END
```

```
      FUNCTION WSP(T)
C--------------------------------------- Franco Stravisi 1988
C  Goff-Gratch definition of saturation water vap pressure:
C  WSP    saturation water vapor pressure / hPa
C  T      air temperature /°C
C------------------------------------------------------------
      TK=T+273.15
      T1=273.16
      T2=1.-T1/TK
      T3=TK/T1
      WSP=10.**(10.79574*T2-5.02800*ALOG10(T3)+0.78614
     .             +1.50475E-4*(1.-10.**(-8.2969*(T3-1.)))
     .             +4.2873E-4*(10.**(4.76955*T2)-1.))
      RETURN
      END
```

```
      PROGRAM AIRCOMP
C---------------------------------------------------
C  Equation of state of moist air
C  Computations through Subroutine AIRES
C---------------------------------------------------
      COMMON/VIRTEM/TV,TVP
      WRITE(*,100)
  100 FORMAT(/
     .' Equation of state of moist air computations.'/
     .' Subroutine AIRES     (Franco Stravisi 1988).'/)
 1000 CONTINUE
      WRITE(*,1)
    1 FORMAT(' p /hPa ? '\)
      READ(*,2) P
    2 FORMAT(F15.6)
      WRITE(*,3)
    3 FORMAT(' t /°C  ? '\)
      READ(*,2) T
      WRITE(*,6)
      TW=0.
      U=0.
    6 FORMAT(' K (1/2 input TW/U) ? '\)
      READ(*,7) K
    7 FORMAT(I5)
      IF(K.EQ.1) WRITE(*,4)
    4 FORMAT(' tw /°C ? '\)
      IF(K.EQ.1) READ(*,2) TW
      IF(K.EQ.2) WRITE(*,5)
    5 FORMAT(' U  / % ? '\)
      IF(K.EQ.2) READ(*,2) U
      CALL AIRES(P,T,TW,U,K,E,EW,R,Q,D,RO)
      WRITE(*,10) P,T,TV,TVP
   10 FORMAT(/
     .' Atmospheric pressure   :',F15.5,' hPa'/
     .' air temperature        :',F15.5,' °C'/
     .' virtual temperature    :',F15.5,'  K'/
     .' adjusted virtual temp. :',F15.5,'  K')
      IF(K.EQ.1) WRITE(*,11) TW
   11 FORMAT(
     .' wet bulb temperature   :',F15.5,' °C')
      WRITE(*,12) U
   12 FORMAT(
     .' relative humidity      :',F15.5,'  %')
      WRITE(*,13) E,EW,R,Q,D,RO
   13 FORMAT(
     .' vapour pressure        :',F15.5,' hPa'/
     .' vapour pressure at sat.:',F15.5,' hPa'/
     .' mixing ratio           :',F15.5,' /1000'/
     .' specific humidity      :',F15.5,' /1000'/
     .' absolute humidity      :',F15.5,'  g/m3'/
     .' air density            :',F15.5,' kg/m3'//)
      GOTO 1000
      STOP
      END
```